

Q: How to set temperature coefficients for temperature-dependent materials?

A: There are two methods available:

- Representing elements of the elasticity matrix with a cubic formula in temperature using variables.
- Representing coefficients of linear thermal expansion with a cubic formula in temperature and entering the calculated values into a table.

Please refer to the next slides.

Temperature-Dependent Elasticity Property

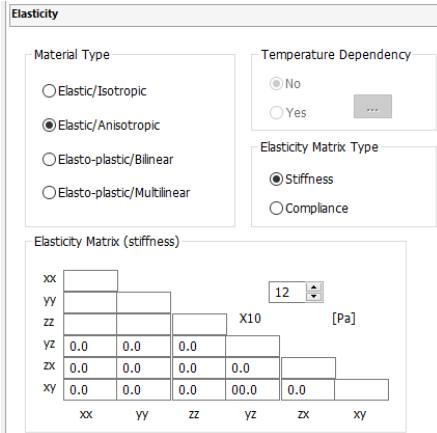
Described below is how to enter elements of the elasticity matrix where those elements are represented by a cubic formula in temperature, shown as formula (1).

$$c_{ij}(T) = c_{ij}(T_0) \left[1 + c_{ij_1}(T - T_0) + c_{ij_2}(T - T_0)^2 + c_{ij_3}(T - T_0)^3 \right] \quad (1)$$

where

c_{ij_1} , c_{ij_2} , c_{ij_3} : First, second, and third degree coefficients for each element of the elasticity matrix.

T_0 : The reference temperature where those coefficients are determined.



Elasticity

Material Type

- Elastic/Isotropic
- Elastic/Anisotropic
- Elasto-plastic/Bilinear
- Elasto-plastic/Multilinear

Temperature Dependency

- No
- Yes

Elasticity Matrix Type

- Stiffness
- Compliance

Elasticity Matrix (stiffness)

xx						
yy						
zz						
yz	0.0	0.0	0.0			
zx	0.0	0.0	0.0	0.0		
xy	0.0	0.0	0.0	0.0	0.0	
	xx	yy	zz	yz	zx	xy

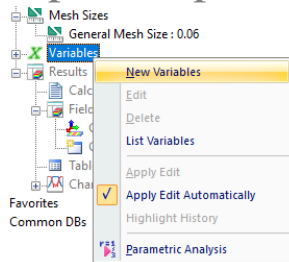
X10 [Pa]

12

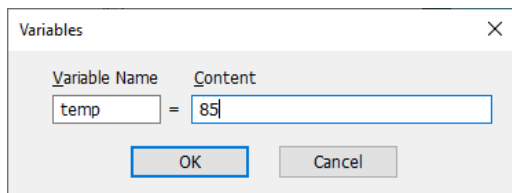
The elasticity matrix, or stiffness matrix, of Femtet can not define temperature dependence. Input the values calculated by the formula (1) into the elasticity matrix.

The variable function of Femtet allows you to change and check the values of the variables through the variable table.

How to input temperature coefficients of materials

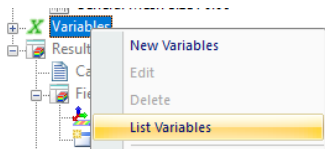


Click on [Variable] and select [New Variables] on the right-click menu.



Input the variable name and the content: a value or expression.

The variables defined before are available.



[List Variables] is selected to display the list. The values and expressions can be modified.

Input Example: Temperature-Dependent Elasticity Matrix

$$c_{ij}(T) = c_{ij}(T_0) \left[1 + c_{ij_1}(T - T_0) + c_{ij_2}(T - T_0)^2 + c_{ij_3}(T - T_0)^3 \right] \quad (1)$$

Setting of c11 and c12

temp = Reached temperature

temp0 = Reference temperature (the reference temperature not for the analysis but for determining the coefficients)

dt = temp – temp0

c11 = c11_0*(1.0+c11_1*dt+c11_2*dt*dt+c11_3*dt*dt*dt)

c12 = c12_0*(1.0+c12_1*dt+c12_2*dt*dt+c12_3*dt*dt*dt)

Elasticity

Material Type

Elastic/Isotropic

Elastic/Anisotropic

Elasto-plastic/Bilinear

Elasto-plastic/Multilinear

Temperature Dependency

No

Yes

Elasticity Matrix (stiffness)

xx	C11				
yy	C12				11
zz	X10				
yz	0.0	0.0	0.0		
zx	0.0	0.0	0.0	0.0	
xy	0.0	0.0	0.0	0.0	0.0
	xx	yy	zz	yz	zx

Elasticity Matrix Type

Stiffness

Compliance

Variables Table

Variable name	Value	Expression
temp0	50.0	50
temp	85.0	85
dt	35.0	temp-temp0
c11_0	0.8605	0.8605
c11_1	-0.0000485	-48.5e-6
c11_2	-0.000000075	-75e-9
c11_3	-0.000000000...	-15e-12
c12_0	0.0505	0.0505
c12_1	-0.002703	-2703e-6
c12_2	-0.0000015	-1500e-9
c12_3	0.00000000191	1910e-12
c11	0.8589596894...	c11_0*(1.0+c11_1*dt+c11_2*dt*dt+c11_3*dt*dt*dt)
c12	0.0456337892...	c12_0*(1.0+c12_1*dt+c12_2*dt*dt+c12_3*dt*dt*dt)

Temperature-Dependent Coefficient of Linear Thermal Expansion

Described below is how to calculate the coefficient of linear thermal expansion where strains are represented by a cubic formula in temperature, shown as formula (1).

$$(L_1 - L_0) / L_0 = \beta_1(\theta_1 - \theta_0) + \beta_2(\theta_1 - \theta_0)^2 + \beta_3(\theta_1 - \theta_0)^3 \quad (1)$$

where

L_0, L_1 : length at θ_1 and θ_2 , respectively.

The coefficient of linear thermal expansion, α , is represented by a quadratic formula.

$$\alpha = \alpha_1 + \alpha_2(\theta - \theta_0) + \alpha_3(\theta - \theta_0)^2 \quad (2)$$

The relationship between L_0 and L_1 is determined as shown below.

$$\begin{aligned} (L_1 - L_0) / L_0 &= \int_{\theta_0}^{\theta_1} \alpha d\theta \\ &= \int_{\theta_0}^{\theta_1} [\alpha_1 + \alpha_2(\theta - \theta_0) + \alpha_3(\theta - \theta_0)^2] d\theta \\ &= \alpha_1(\theta_1 - \theta_0) + \frac{1}{2}\alpha_2(\theta_1 - \theta_0)^2 + \frac{1}{3}\alpha_3(\theta_1 - \theta_0)^3 \end{aligned} \quad (3)$$

Formulas (1) and (2) give the following formula.

$$\alpha_1 = \beta_1, \alpha_2 = 2\beta_2, \alpha_3 = 3\beta_3 \quad (4)$$

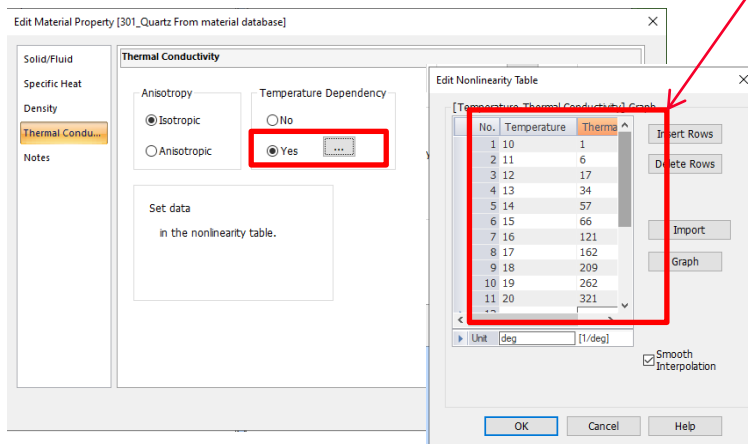
How to Set the Coefficient of Linear Thermal Expansion

Where $\beta_1 = \beta_2 = \beta_3 = 10^{-6}$ and $\theta_1 = 10^\circ\text{C}$, $\theta_2 = 20^\circ\text{C}$ are given,

Formula (4) gives the coefficient of linear thermal expansion, α , as shown below.

$$\alpha = [1 + 2(\theta - \theta_0) + 3(\theta - \theta_0)^2] \times 10^{-6}$$

Input the calculated values at proper intervals.



The screenshot shows two overlapping dialog boxes. The background box is 'Edit Material Property [301_Quartz From material database]' with 'Thermal Conductivity' selected. The foreground box is 'Edit Nonlinearity Table' with a table of data. A red box highlights the 'Yes' radio button in the 'Temperature Dependency' section of the background dialog and the table in the foreground dialog. A red arrow points from the equation above to the table.

No.	Temperature	Thermal Conductivity
1	10	1
2	11	6
3	12	17
4	13	34
5	14	57
6	15	66
7	16	121
8	17	162
9	18	209
10	19	262
11	20	321

Results

Theoretical Value

$$L_1 - L_0 = L_0[(20 - 10) + (20 - 10)^2 + (20 - 10)^3] \times 10^{-6} = 1.110 \times 10^{-5}$$

Analysis Model

